CHURCHLANDS SENIOR HIGH SCHOOL
MATHEMATICS SPECIALIST 3, 4 Last Test 2016
NON-Calculator Section
Chapters 9 to 12

Name_____

Time: 20 minutes
Total: 22 marks

1 [14 marks]

(a) Determine the indefinite integrals.

(i) $\int 2x\,(4x^2-3)^4\,dx$ [2 marks]

(ii) $\int \sin^4 x\,dx$ [4 marks]

(b) Use the substitution $u = e^x + 3$ to find $\int \frac{e^{2x}}{e^x + 3}\, dx$. [4 marks]

(c) $\int cos\ 2x\ sin^3 2x\ \ dx$ [2]

2 [7 marks]

Show that the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ around the $y-$ axis is $\frac{96\pi}{5}$ [4 marks]

3 [3 marks]

Find the general solution to the differential equation $\frac{dy}{dx} = xy(x+3)$

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Name_____

Time: 35 minutes
Total: 30 marks

4. [11 marks]

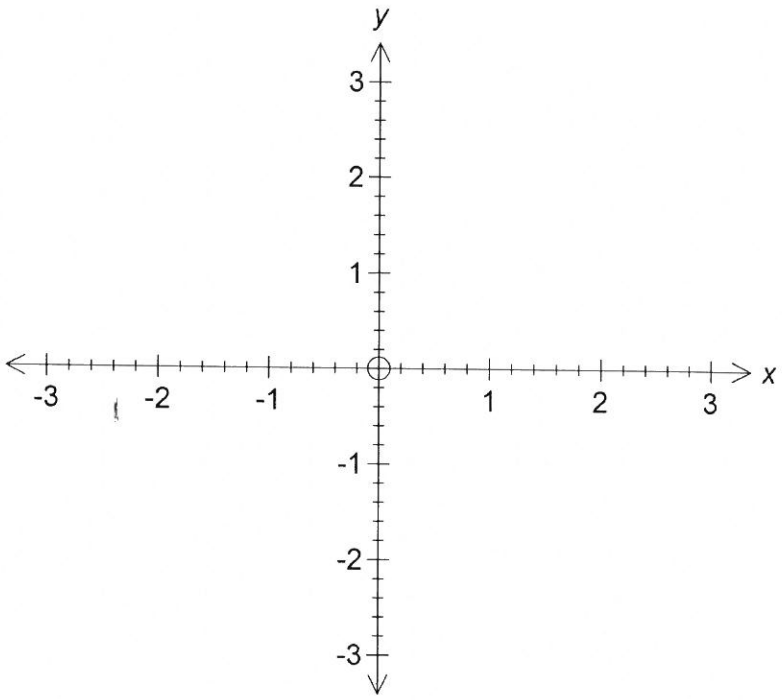
(a) The equation of motion of a body is given by $\frac{d^2x}{dt^2} + 16x = 0$, where x is the distance of the body (in cm) to a fixed point O and t is time in seconds. It is known that the body starts moving from O with a velocity of -1 cm s^{-1}. Show clearly that $x = A\sin(\omega t + \alpha)$ giving the values of A, ω (where $\omega > 0$) and α. [6]

(b) The equation of motion of another body is given by $x = 2 \cos (0.5\, t)$.the average speed of the body in the first ten seconds. .Show clearly how you obtained your answer.

[5]

5 [4 marks]

Refer to the differential equation $\frac{dy}{dx} = 2x$. Determine the specific solution of the differential equation that passes through (1, 1). Graph the slope field below and graph the solution too.



6. [7 marks]

A randomly selected group of 20 Year 12 girls have a mean height of 160 cm with a standard deviation of 1.5 cm.

(a) Use the data to predict the height and standard deviation of all WA Year 12 girls. (3)

(b) State the 95% confidence limits for the mean height of Year 12 girls in WA. (2)

(c) Explain the difference between 95% confidence limits and 90% confidence limits. (2)

7. [8 marks]

The number of deaths from Avian (bird) Influenza in Cambodia was reported to the World Health Organisation (WHO).
Over the years 2010-2015 the number of deaths from Avian Influenza can be determined by the equation $N = \frac{8}{1+128.866e^{-3.529t}}$ where t is in years where $t = 0$ in 2010.

(a) Determine the number of deaths in Cambodia from Avian Influenza in 2012. (2)

(b) (i) Find the rate of increase of cases of Avian Influenza in 2015. (3)

(ii) Is the number of cases increasing or decreasing in 2013? (1)

(c) Sketch the shape of the curve (2)

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NON-Calculator Section
Chapters 9 to 12

Name SOLUTIONS

Time: 20 minutes
Total: 22 marks

1 [14 marks]

(a) Determine the indefinite integrals.

(i) $\int 2x\,(4x^2-3)^4\,dx$ [2 marks]

$= \frac{1}{4}\,\frac{(4x^2-3)^5}{5} + c$ ✓✓

(ii) $\int \sin^4 x\,dx = \int \sin^2 x \sin^2 x\,dx$ [4 marks]

$= \int \frac{(1-\cos 2x)}{2}\frac{(1-\cos 2x)}{2}\,dx$ ✓

$= \frac{1}{4}\int 1 + \cos^2 2x - 2\cos 2x\,dx$

$= \frac{1}{4}\int 1 + \frac{1+\cos 4x}{2} - 2\cos 2x\,dx$ ✓

$= \frac{1}{8}\int 3 + \cos 4x - 4\cos 2x\,dx$ ✓

$= \frac{3x}{8} + \frac{\sin 4x}{32} - \frac{\sin 2x}{4} + c$ ✓

(b) Use the substitution $u = e^x + 3$ to find $\int \frac{e^{2x}}{e^x + 3}\, dx$. [4 marks]

$u = e^x + 3$

$dx = \frac{1}{e^x}\, du$ ✓

$$\int \frac{e^{2x}}{e^x + 3}\, dx = \int \frac{e^{2x}}{u} \frac{1}{e^x}\, du$$

$$= \int \frac{e^x}{u}\, du \quad ✓$$

$$= \int \frac{u - 3}{u}\, du$$

$$= \int 1 - \frac{3}{u}\, du \quad ✓$$

$$= u - 3\ln u + c \quad ✓$$

$$= e^x + 3 - 3\ln(e^x + 3) + c \quad ✓$$

(c) $\int \cos 2x \sin^3 2x\, dx$ [2]

$u = \sin 2x \quad \frac{du}{dx} = 2\cos 2x$

$\Rightarrow \quad dx = \frac{du}{2\cos 2x}$ ✓

$$\int \cos 2x \sin^3 2x\, dx$$

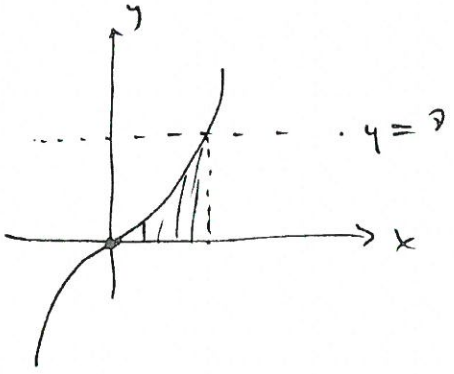
$$= \int \frac{u^3}{2}\, du$$

$$= \frac{u^4}{8} + c$$

$$= \frac{\sin^4 2x}{8} + c \quad ✓$$

2 [7 marks]

Show that the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ around the $y-$ axis is $\frac{96\pi}{5}$ [4 marks]



$y = 8 : x = 2$

$y = x^3$

$\Rightarrow x = y^{1/3}$ ✓

$\int_0^8 \pi x^2 dy = \int_0^8 \pi \left(y^{1/3}\right)^2 dy$ ✓

$= \pi \left[\frac{3}{5} y^{5/3}\right]_0^8$ ✓

$= \pi \, 8^{5/3} \times \frac{3}{5}$

$= \pi \cdot 32 \cdot \frac{3}{5}$

$= \frac{96}{5}\pi$ ✓

3 [3 marks]

Find the general solution to the differential equation $\frac{dy}{dx} = xy(x+3)$

$\frac{dy}{y} = (x^2 + 3x)\,dx$ ✓

$\ln|y| = \frac{x^3}{3} + \frac{3x^2}{2} + c$

✓ ✓

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Name _____ **Time: 35 minutes**
Total: 30 marks

4. [11 marks]

(a) The equation of motion of a body is given by $\frac{d^2x}{dt^2} + 16x = 0$, where x is the distance of the body (in cm) to a fixed point O and t is time in seconds. It is known that the body starts moving from O with a velocity of -1 cm s^{-1}. Show clearly that $x = A\sin(\omega t + \alpha)$ giving the values of A, ω (where $\omega > 0$) and α. [6]

$\frac{d^2x}{dt^2} = -16x$

$= -4x^2$ ✓

$\therefore$ SHM with ang vel $\omega = 4$

$\therefore x = A\sin(4t + \alpha)$ ✓

when $t = 0$, $x = 0$ $\therefore 0 = A\sin\alpha$

$\Rightarrow \alpha = 0$ ✓

Vel $= v = \frac{dx}{dt} = 4\cos 4t$ ✓

$t = 0$, $v = -1$, $\Rightarrow 4A = -1$ $\therefore A = -\frac{1}{4}$ ✓

$\therefore x = -\frac{1}{4}\sin 4t$ ✓

* The $x = \frac{1}{4}\sin(4t + \pi)$ is acceptable too.

$\therefore \alpha = \pi$

hence $A = 1/4$ as $|4A| = 1$

$4A = 1$

$A = 1/4$

(b) The equation of motion of another body is given by $x = 2\cos(0.5t)$. the average speed of the body in the first ten seconds. .Show clearly how you obtained your answer. [5]

$$\text{Dist} = \int_0^{10} \left| -\sin\frac{t}{2} \right| dt \quad \checkmark\checkmark$$

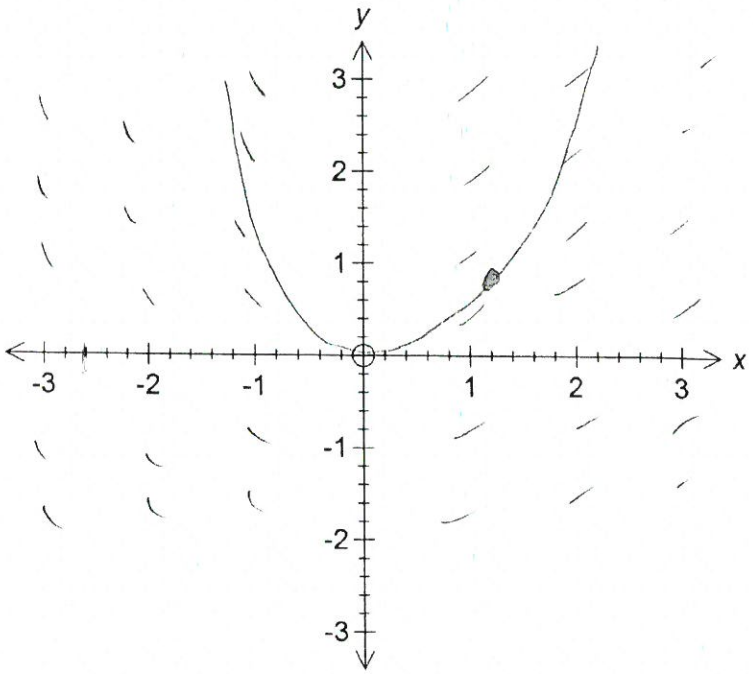
$$= 6.567 \quad \checkmark$$

$$\therefore \text{Av speed} = \frac{6.567}{10}$$

$$= 0.657 \text{ m/s.} \quad \checkmark\checkmark$$

5 [4 marks]

Refer to the differential equation $\frac{dy}{dx} = 2x$. Determine the specific solution of the differential equation that passes through (1, 1). Graph the slope field below and graph the solution too.



✓✓ slope field
✓ Graph
✓ Point

6. [7 marks]

A randomly selected group of 20 Year 12 girls have a mean height of 160 cm with a standard deviation of 1.5 cm.

(a) Use the data to predict the height and standard deviation of all WA Year 12 girls. (3)

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ Expected mean = 160 cm ✓

$1.5 \times \sqrt{20} = \sigma \therefore \sigma \approx 6.7$ cm. ✓

Assume means are equal.

If answer $\sigma = 1.5$ or other ✗ then FTE.

(b) State the 95% confidence limits for the mean height of Year 12 girls in WA. (2)

95% confidence $\therefore z = 1.96$

$160 \pm 1.96 \times 1.5 \times \sqrt{20}$

$= 160 \pm 13.15$

$146.85 \leq \mu \leq 173.15$

$160 - \frac{1.6 \times 15}{\sqrt{20}} \leq \mu \leq 160 + \frac{1.6 \times 15}{\sqrt{20}}$

$159.34 \leq \mu \leq 160.66$

$159.72 \leq \mu \leq 160$

(c) Explain the difference between 95% confidence limits and 90% confidence limits. (2)

To a 90% sure the confidence limits are. closed, you have a smaller range as there is a 10% error margin. ✓

To make ever more certain a 95% confidence limit there or will have a wider range so that there is a larger chance that the mean is included ✓

7. [8 marks]

The number of deaths from Avian (bird) Influenza in Cambodia was reported to the World Health Organisation (WHO).
Over the years 2010-2015 the number of deaths from Avian Influenza can be determined by the equation $N = \frac{8}{1+128.866e^{-3.529t}}$ where t is in years where $t = 0$ in 2010.

(a) Determine the number of deaths in Cambodia from Avian Influenza in 2012. (2)

$N(2) = \frac{8}{1+128.866e^{-3.529t}} = 8$ ✓

$\frac{dN}{dx} = -8\left(1+128.866e^{-3.529t}\right)^{-2} \times \left(0+128.866e^{-3.529t} \times (-3.529)\right)$

$dN/dx \Big|_{t=5} = 0.000074 \simeq 0$ ✓

(b) (i) Find the rate of increase of cases of Avian Influenza in 2015. (3)

$\frac{dN}{dx} = \frac{3638.144912e^{-3.529t}}{\left(1+128.866e^{-3.529t}\right)^2}$

At $t = 3$ $\frac{dN}{dx} = 0.09123$

+ve so increasing.

(ii) Is the number of cases increasing or decreasing in 2013? (1)

+ve + increasing.

(c) Sketch the shape of the curve (2)